PRN:		Course Code :	USS403

Second Year **B. Sc.** End Semester Examination Summer 2024

Mathematics -IV

Day and Date: 27 MAY 2024, Mon Max Marks: **100**

Time : 02:00PM To 05:00PM Duration: **03** hrs

Instructions: All questions are compulsory.
Sub question 'a)' is compulsory, Solve any two from the remaining

		Marks	Bloom's Level	CO
Q.1	Attempt the following			
a)	Show that intersection of two subgroups is again a subgroup.	06	L1	CO1
b)	State and prove Lagrange's theorem.	07	L3	CO1
c)	Prove that a non empty subset H of a group G is a subgroup of G if and only if $a,b \in H \Rightarrow ab^{-1} \in H$.	07	L3	CO1
d)	Show that center of a group G is a subgroup of a group G.	07	L2	CO1
Q.2	Attempt the following			
a)	Define 1) Order of an element, 2) Cyclic group, 3) Euler's totient function.	06	L3	CO2
b)	Show that infinite cyclic group has precisely two generators.	07	L3	CO2
c)	Show that order of cyclic group is equal to order of its generator	07	L3	CO2
d)	Show that a group of prime order is abelian.	07	L1	CO2
Q.3	Attempt the following			
a)	If $f : G \to G'$ be a homomorphism then show that f is one one if and only if $Ker\ f = \{e\}$	06	L3	CO3
b)	Show that intersection of two normal subgroups is again a normal subgroup.	07	L1	CO3

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c) If $f: G \to G'$ be an onto homomorphism with $K = Ker\ f$ then prove that $\frac{G}{K} \cong G'$ — 07 L2 CO3

d) If $f: G \to G'$ be a homomorphism then show that $Ker\ f$ is a normal subgroup of G — 07 L2 CO3

Q.4 Attempt the following

a) Show that intersection of two subring is again a subring. — 06 L3 CO4

b) Show that every field is an integral domain. — 07 L2 CO4

c) Define — 07 L2 CO4

i) Idempotent
ii) Integral domain
iii) Division ring
iv) Nilpotent

d) Show that a non-empty subset S of ring R is a subring of R if and only if $a, b \in S \Rightarrow a - b \in S,\ a \cdot b \in S$. — 07 L2 CO4

Q.5 Attempt the following

a) Show that $Ker\ f = \{0\}$ if and only if f is one one. — 06 L1 CO5

b) If A and B are two ideals of ring R then prove that $A + B$ is an ideal of R. — 07 L2 CO5

c) Let A, B be two ideals of a ring R then show that $\frac{A+B}{A} \cong \frac{B}{A \cap B}$ — 07 L1 CO5

d) If $f: R \to R'$ be a homomorphism then show that — 07 L2 CO5

1) $f(0) = 0'$
2) $f(-a) = -f(a)$ Where 0 and $0'$ are zero of R and R' respectively.

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